Busts in House Prices

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To examine the risk of asset prices either for purposes of modeling or portfolio construction, many analysts use the variance of price returns as their risk measure. However, if the return distributions are fat tailed, the variance does not sufficiently assess the real risk faced. Especially when comparing the returns on different assets, this may lead to false conclusions concerning their relative riskiness. In this paper, we examine the distributional characteristics of United States and Dutch house price returns and find that these are distributed with much fatter tails than a normal distribution. We then focus explicitly on this observed tail fatness and analyze the risk on house price indices and stock indices in terms of the probability on extreme events over different time horizons. Our results clearly indicate that the variance as a risk measure underestimates the real risk associated with house price movements and that risk due to extremes is persistent.

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1 Introduction

Consumer wealth is affected strongly by variations in house prices. The typical U.S. family's exposure to the housing market is much stronger than the exposure to the stock market and the total value of the U.S. housing market is much higher than that of the stock market. This implies that variations in house prices could well have a much larger effect on consumer wealth than movements of stock prices. This, in turn, could imply that movements in house prices have a stronger influence on economic growth than movements in stock prices. Unlike equity markets, consumers basically have no instruments to hedge themselves against the risk they face due to house price changes. As argued by Shiller (1993), the absence of such instruments can partly be explained by the lack of knowledge that exist about constructing house price indices econometrically and by stating and measuring the correct perception of home-owning families towards the risk they face on the prices of their homes.

To provide information on the characteristics of house price changes, indices have been constructed that reflect the performance of a portfolio of houses in several urban areas. Example of such indices are found in Case and Shiller (1987, 1989) for several U.S. cities, in Mahieu and van Bussel (1996) for the Netherlands, and in Eichholtz (1997) for the Herengracht, one of the canals in Amsterdam. These studies all present standard deviations as benchmark risk statistics and their results imply that the risk levels (and return levels) are lower for their house price indices than for stock indices over similar time periods and for the same countries. Eichholtz, Koedijk, Nieuwland, and Nissen (1995 EKNN) show that although risk in terms of standard deviations is

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1 See Mishkin (1978) and Shiller (1993) among others.
lower, extreme movements in prices occur more frequently indicating that home owners face more risk on extreme events than is the case with common stock.

In this paper, we extent the EKNN study in two ways. EKNN use Eichholtz' Herengracht Index to test their hypothesis. Although the Herengracht Index covers a long period of time (1628 through 1973), which provides a relative large data-set with biannual returns, it is based on a relative small number of houses which are not traded on a frequent basis. The variability in a repeat sales index is both due to the true variability in house prices and to index estimation errors. This means that the risk of house prices should be examined with an index in which the estimation error is as low as possible. Only indices that are based on many sales observations are suited for this reason. That is why we look at datasets with many cross-sectional observations for two countries. For the United States we use the Case and Shiller data for Boston, Chicago, and Los Angeles and for the Netherlands we use the Mahieu and van Bussel index to compare the results over more than one country\(^2\).

Apart from choosing the appropriate datasets, we improve the measures of exposure to extreme risk used by EKNN. They used the conventional estimator developed by Hill (1975) to assess the tail fatness of the distribution of returns on the Herengracht Index. Simulation studies have shown that this Hill estimator suffers significantly from a small sample bias. Huisman, Koedijk, Kool and Palm (1997 HKKP) recently presented a simple estimator that corrects for the small sample bias in the Hill estimator and we apply their procedure here to obtain accurate measures of the extreme risks that house owners face.

\(^2\) The Herengracht Index consists of 3,623 repeat sales pairs for the period 1623 through 1973. The van Bussel and Mahieu index for the Netherlands is based on 228,144 repeat sales for the period 1973 through 1996.
This paper presents the methodology we employ to assess extreme risk in Section 2. The data used is presented in Section 3. Section 4 contains the results we find. In short, we confirm the statement that standard deviation underestimates the risk faced by home owners and their exposure to extreme price movements of their house is comparable to the same exposure to their investments in common stock, both for the U.S. and the Netherlands. Lastly, Section 5 concludes this research and gives suggestions for future research.

2 Methodology

As discussed in the introduction, we examine the extreme movements in house prices based on several house price indices. We therefore concentrate on the tails of the distributions of returns on house price indices and indices on common stock. If the return distribution on house price is fatter than for common stock, extreme movements in house prices occur more frequently than extreme movements in stock prices. The measure we use to compare the tail fatness over several distributions is the tail index. The tail index stems from the Extreme Value Theory and measures the speed with which the tails of a fat-tailed distribution approaches zero. The lower the speed, the fatter the tail. An important characteristic of the tail index is that it is one-to-one related to the number of existing moments for the distribution under consideration. If the tail index $\alpha$ equals 3 for example, the number of existing moments equals 3. This would imply that distribution characteristics as the mean, the standard deviation and skewedness are finite but that for example kurtosis does not exist. This example directly the advantage of using the tail index to compare tail fatness instead of kurtosis. We do not need to make any assumption regarding the
number of existing moments in the distributions of returns on house prices\(^3\).

Regarding the issue of extreme movements in house prices, we therefore test the hypothesis that the tail index for returns on house prices reflects more tail fatness than the tail index for common stock.

2.1 The Tail Index \(\alpha\)

The tail index \(\alpha\) measures the speed with which the tail of a fat-tailed distribution \(F(.)\) approaches zero, when \(F(.)\) fulfills the following regular variation condition at infinity\(^4\):

\[
\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}. 
\]

(1) implies that the higher \(\alpha\) is the less fat-tailed the distribution is. Because the tail-index equals the maximum number of finite moments in the sample, the tail-index can directly be used to test for the number of existing moments.

In this section, we briefly discuss the HKKP-estimator as a simple and appropriate tool to obtain unbiased tail-index estimates in small samples.\(^5\) The methodology starts from the observation that the bias of the conventional Hill (1975)-estimator is a function of sample size \(n\). Then, the bias-function is used to correct for the small sample bias. In this respect, the HKKP-estimator may be seen as a modified.\(^6\)

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\(^3\) It has been shown for return distributions on several financial assets that the number of existing moments is often smaller than four. This implies that kurtosis can not be used in these cases to compare tail fatness over different distributions. We refer to Jansen and de Vries (1991), Loretan and Phillips (1994), and Huisman, Koedijk, Kool and Palm (1997, 1998) for tail index estimates for returns on exchange rates and common stock.

\(^4\) Many studies have shown that many financial series fulfill this condition.

\(^5\) This estimator is discussed and tested in a more elaborate way by HKKP.

\(^6\) In the literature, the tail-index is referred to as \(\alpha\) either or \(\gamma\), where \(\alpha\) equals \(1/\gamma\); we use both interchangeably.
2.2 The Hill (1975) estimator

Suppose a sample of $n$ observations is drawn from some unknown fat-tailed distribution. Let the parameter $\gamma$ be the tail-index of this distribution (per definition, $\gamma$ equals $1/\alpha$, where $\alpha$ refers to the maximum number of existing finite moments), while $x_i$ is the $i^{th}$ increasing order statistic ($i = 1..n$). Hill (1975) proposes the following estimator for the tail-index:

$$\gamma(k) = \frac{1}{k} \sum_{j=1}^{k} \ln(x_{n-j+1}) - \ln(x_{n-k}),$$

where $k$ is the pre-specified number of tail observations to include ($k = 1 .. n-1$). The choice of $k$ is crucial to obtain good -- that is, unbiased -- estimates of the tail-index. In this respect, an asymptotic approximation of the bias in the Hill estimator can be derived for the following class of distribution functions:

$$F(x) = 1 - ax^{-\alpha} (1 + bx^{-\beta}),$$

where $\alpha, \beta > 0$ and $a$ and $b$ are real numbers. It has been shown that for a wide range of fat-tailed distribution equation (3) provides the second order asymptotic expansion of the cumulative distribution function (c.d.f.). For this class of distribution functions, the expected value and the variance of the Hill estimator in large samples for a given $k$ can be approximated by

$$E(\gamma(k)) \approx \frac{1}{\alpha} - \frac{b\beta}{\alpha(\alpha + \beta)} a^{-\alpha} \left( \frac{\beta}{n} \right)^{\beta},$$

and

$$\text{var}(\gamma(k)) \approx \frac{1}{k\alpha^2},$$
respectively. According to equation (4) and (5), the bias increases with \( k \), while the variance increases. A trade-off results between unbiasedness and accuracy. Most empirical studies suffer from this trade-off problem. Generally, a single 'optimal' \( k \) is selected. However, equation (4) clearly shows that a bias exists for any \( k \) exceeding zero. One way out of the dilemma is to increase the number of observations \( n \), which decreases the importance of the bias for any given \( k \) decreases.

2.3 The HKKP estimator

The HKKP estimator which is presented here, overcomes the problem of the need to select an optimal \( k \) in small samples, by exploiting an important characteristic of the bias function. For values of \( k \) smaller than some threshold value \( \kappa \) (for example for \( k \leq n/2 \)), we show that the bias of the conventional Hill-estimate of \( \gamma \) increases almost linearly in \( k \). Rewriting equation (4) with the bias-term denoted by \( f(k) \) leads to:

\[
\gamma(k) = \beta_0 + f(k) + \varepsilon(k),
\]

where \( \varepsilon(k) \) is an idiosyncratic error term. Due to the fact that the bias in the Hill estimate can be approximated by a linear function of \( k \), for \( k \leq \kappa \), equation (6) can be approximated by:

\[
\gamma(k) = \beta_0 + \beta_1 k + \varepsilon(k), \quad k = 1 \ldots \kappa.
\]

Instead of selecting a single value of \( k \) to estimate the tail-index of the distribution under consideration, we propose to compute \( \gamma(k) \) for a range of values of \( k \) from 1 to \( \kappa \). Subsequently, the vector of computed \( \gamma(k) \)'s is used in equation (7). In subsection 0, we already argued that an unbiased estimate of \( \gamma \) can be obtained only for \( k \)

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7 Various studies select \( k \) to minimize the MSE of the Hill-estimates conditional on a known distribution,; see for instance Koedijk and Kool (1994), and Loretan and Phillips (1994).
approaching 0. Evaluation of equation (7) for k approaching zero, yields an optimal unbiased estimate of $\gamma$ equal to the estimated intercept $\beta_0$. Applying this procedure circumvents the bias-variance trade-off, since we use the information from a whole range of conventional Hill estimates for the different values of k to obtain an estimate for the tail-index.

2.3.1 Econometric considerations

Although the coefficients in (7) can be estimated by Ordinary Least Squares, two issues complicate the procedure. First, equation (5) indicates that the variance of Hill estimates $\gamma(k)$ varies with k, so that the error term $\epsilon(k)$ in equation (7) is heteroskedastic. As an alternative, HKKP suggest a Weighted Least Squares (WLS) approach to correct for this form of heteroskedasticity. Second, an overlapping data problem exists due to the construction of $\gamma(k)$. The variables $\gamma(k)$ are correlated, in terms of k, since estimates $\gamma(k)$ and $\gamma(m)$ where $k \neq m$ are based on $1+\min(k,m)$ common observations, see equation (2). Consequently, the usual formulae for the standard errors are inappropriate both for the estimates. Given the complexity, we refer to the HKKP paper for information on calculating the correct standard errors.

2.3.2 Point estimates

To obtain the tail-index formulae, we first write (7) in the following matrix notation

$$\gamma = Z\beta + \epsilon,$$

where $Z$ is a ($\kappa \times 2$) matrix with ones in the first column and the vector $\{1, 2, ..., \kappa\}'$ in the second. Equation (5) reveals that the variance of the Hill estimator is inversely related to k. To correct for this form of heteroskedasticity, we propose to apply WLS with a ($\kappa \times \kappa$) weighting matrix W. Given the variance specification, W has $\{\sqrt{1}, \sqrt{2}$,
\[ \sqrt{\kappa} \] as diagonal elements and zeros elsewhere. Transformation of equation (8) through pre-multiplication with matrix W then results in:

(9) \[ W\gamma = WZ\beta + W\epsilon, \]

in which the error \( W\epsilon \) is homoskedastic. We then obtain the following WLS estimator for the elements in the vector \( \beta \):

(10) \[ b_{WLS} = (Z'W'WZ)^{-1}Z'W'W\gamma, \]

The estimated tail-index \( \gamma \) equals the first element of the vector \( b_{WLS} \).

3 Data

To examine the extreme movements in house prices indices, we selected data that are based on as many cross-sectional observations as possible. For the United States we use weighted repeat sales indices constructed by Case Shiller Weiss Inc. for Boston, Chicago, and Los Angeles as for these cities a sufficiently long time horizon was available\(^8\). We refer to the Case and Shiller (1987, 1989) papers for more information. In order to obtain insight in the stability of the results over different countries we also perform a similar analysis to data obtained for the Netherlands. We use the weighted repeat sales index constructed by Mahieu and van Bussel (1996). This index is based on data provided by the Dutch association of real estate agents (NVM) and the complete data set contains 57% of all transactions on the Dutch residential property market. After filtering, the weighted repeat sales index is built on 228,144 repeat sales pairs. The index covers 272 months from May 1973 through August 1995.

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\(^8\) The house price indices for Boston starts in January 1982, for Chicago in January 1980, and for Los Angeles in July 1970. The last observation for all indices was in March 1998.
We compare the extreme movements in house prices with similar movements in equity portfolios, given our hypothesis that relates the risks of owning a home to the risk derived from a stock portfolio. As the weighted repeat sales indices that we use reflect prices and do not compound dividend compensation, we use stock price indices to reflect movements in the equity portfolios. We use the S&P 500 Composite Price Index for the United States and the CSB General Price Index for the Netherlands\(^9\).

4 Results

In this section we discuss the results of our investigation to the risks faced on movements in house prices for the United States and the Netherlands. In analyzing the risk characteristics for house prices in both countries, we concentrate us on both standard deviations and the tail index as risk measures and compare these with values obtained from common stock indices.

4.1 United States

We examine the house price indices for three major cities in the United States: Boston, Chicago and Los Angeles. From these indices, we calculate the risk measures standard deviation and the tail-index in order to compare these with similar measures for a portfolio consisting of S&P 500 stocks. The stock indices match the time horizon of the house price indices for each city. The results are presented in table 1.

\[ \text{\ldots insert Table 1 approximately here \ldots} \]

\[ \text{\ldots insert Table 1 approximately here \ldots} \]

\[^9\text{The indices are obtained from Datastream.}\]
Table 1 reveals first that the standard deviations for the stock portfolio roughly triples the standard deviations of all cities indices. This is not a striking result as it is widely known that changes in house prices are less variable than changes in stock prices. House prices are thus less risky in terms of standard deviation than common stock, which is reflected in the lower average returns.

The standard deviation reflects the structure of the center of a return distribution. We have argued that home owners face both risk from the center from a return distribution as from extreme movements, i.e. the tails, and if we focus on the latter risk we find a result different than the one described previously. The $\gamma$ tail-index estimates for house prices equal 0.18 for Boston, 0.15 for Chicago and 0.24 for Los Angeles which has by far the longest horizon. In $\alpha$ terms, they hover between 4 and 6 which implies that at least the first four moments exist for house price changes. We find the stock portfolio’s $\gamma$ tail index varying between 0.24 and 0.29 due to different time horizons. Compared to house prices, the tail-indices for stock portfolios are comparable in size. This holds in particular for the Los Angeles index with the longest horizon which has a $\gamma$ estimate of 0.24 for house prices compared to a $\gamma$ estimate of 0.29 for stock. The estimates are not significantly different from each other. This leads to the conclusion that there are not much differences between the tail-indexes of homes and stock in the United States. The exposure to extreme movements is comparable. Although the standard deviation of house price changes is three times as low, the risk from the tail of the distribution is the same. The statement that a home investment is less risky than an investment in common stock does only make sense in terms of standard deviation; not in terms of extreme risks. Busts in house prices occur as frequently as busts in common stock.
4.2 The Netherlands

The Dutch situation is analyzed by comparing the Mahieu and van Bussel repeat sales index with the general price return index of the Dutch stock market: the CBS general price index. From both indices, we calculate returns as the difference between the log values of successive monthly index values. Table 2 presents return statistics and tail index estimates for both indices. The tail indices reflect the fatness of both tails simultaneously; the tail indices are based on the absolute values of the monthly returns in excess of their average return.

… insert Table 2 approximately here …

Regarding the standard deviations, we observe from Table 2 that the monthly returns on Dutch house price index have a standard error equal to 1.29% over the period 1973 through 1995. Over the same era, monthly returns on Dutch common stock showed a standard deviation equal to 4.75%.

The tail index estimate for the Dutch house prices equal 0.35 in $\gamma$ terms which is 2.82 in $\alpha$ terms\(^{10}\). From the second column in Table 2, we learn that the tail index estimate for movements in Dutch stock prices is roughly the same; 0.33 in $\gamma$ terms which is 3.05 in $\alpha$ terms. Given the values of the standard errors, both tail index estimates do not differ significantly. We thus may conclude that return on house prices and common stock are almost equally fat tailed for the Dutch situation; the frequency with which large movements in prices occur is roughly the same for both.

\(^{10}\) The $\alpha$ estimate of almost 3 rejects the hypothesis that the 4\(^{th}\) moment exists for the distribution of returns on Dutch house prices and therefore justifies our approach for comparing the tail structure.
distributions. Although people face less risk on their homes than on their stock portfolio in terms of standard deviation, they do face a similar risk in terms of extreme movements.

5 Concluding Remarks

In this paper, we compare the risks that house owners face from movements in the value of their homes risk and from movements in their common stock investments. Using stock and house price indices for the United States and the Netherlands, we find that the standard deviation of house price changes is about three times lower than that of stock price changes. This huge differences agrees with the existing literature regarding house price risk.

However, to assess the risk individuals face from their homes and their stock portfolio, we do not only use the conventional standard deviation as a risk measure, but we also concentrate on the exposure to extreme price movements faced by stock- and home-owners. Technically we split the risk faced from the center and the tails of both return distributions. We measure the latter risk exposure by calculating the tail index of the asset price changes using a new methodology developed by Huisman, Koedijk, Kool and Palm (1997). The results of this analysis show that the risk faced from extreme price changes is comparable for houses and common stocks. Since conventional risk measures are in terms of standard deviations, the true risk faced from changes in house prices are underestimate relative to stock ownership.

The question is, of course, whether the tail index is a more appropriate measure for risk than the standard deviation. We think it is, at least partly. Since house price

using tail indices instead of kurtosis measures for which we need to assume that the 4th moment is finite.
changes are fat-tailed like the returns on many other financial assets, extreme returns occur frequently and the standard deviation is not a theoretically perfect measure to assess to completed risk faced. The tail index is relevant, since the risk of a few extreme price changes has probably more economic meaning to an investor or homeowner than the risk deriving from a lot of small price changes, and the standard deviation does not distinguish between extreme and small risk.

The high extreme risk related to owning a home is all the more relevant given the amounts at stake for typical households, and given the way households typically finance their home. Whereas buying stocks with borrowed money is not very common, doing the same thing with ones house is standard practice. Our results indicate that this strategy is associated with more risk than has been known previously. This underlines the importance of instruments to manage this risk, be it insurance products as proposed by Shiller and Weiss (1994), derivatives as proposed by Case and Shiller (1993), or risk sharing devices as proposed by Geltner, Miller and Snavely (1995) and Caplin, Chan, Freeman and Tracy (1997) and stresses the importance of tail fatness and the tail-index in this context.
6 References


1 Results for The United States

This table presents the return and risk statistics based on the Case and Shiller weighted repeated sales index for house prices in the United States. The S&P 500 Composite Price Index is taken as the general stock market index for the United States. We analyze the monthly log returns on both the house price indices (Homes) and the S&P 500 (Stock). The house price indices for the three cities start in January 1982 for Boston, in January 1980 for Chicago, and in July 1970 for Los Angeles and all end in March 1998. The results for the stock indices are built on the data for the same period of time as the house price indices.

<table>
<thead>
<tr>
<th></th>
<th>Boston</th>
<th></th>
<th>Chicago</th>
<th></th>
<th>Los Angeles</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Homes</td>
<td>Stock</td>
<td>Homes</td>
<td>Stock</td>
<td>Homes</td>
<td>Stock</td>
</tr>
<tr>
<td>n</td>
<td>194</td>
<td>218</td>
<td>333</td>
<td>333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.55</td>
<td>1.14</td>
<td>0.41</td>
<td>1.04</td>
<td>0.58</td>
<td>0.80</td>
</tr>
<tr>
<td>standard deviation</td>
<td>1.64</td>
<td>4.23</td>
<td>1.24</td>
<td>4.26</td>
<td>1.57</td>
<td>4.33</td>
</tr>
</tbody>
</table>

### tail estimates

<table>
<thead>
<tr>
<th></th>
<th>Boston</th>
<th></th>
<th>Chicago</th>
<th></th>
<th>Los Angeles</th>
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<tbody>
<tr>
<td></td>
<td>Homes</td>
<td>Stock</td>
<td>Homes</td>
<td>Stock</td>
<td>Homes</td>
<td>Stock</td>
</tr>
<tr>
<td>γ</td>
<td>0.18</td>
<td>0.24</td>
<td>0.15</td>
<td>0.24</td>
<td>0.24</td>
<td>0.29</td>
</tr>
<tr>
<td>s.e. (γ)</td>
<td>0.04</td>
<td>0.06</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>α</td>
<td>5.66</td>
<td>4.23</td>
<td>6.62</td>
<td>4.11</td>
<td>4.25</td>
<td>3.42</td>
</tr>
</tbody>
</table>

The mean returns and the standard deviations on the returns are percentages based on monthly differences between successive log values of the indices. The number of returns used is referred to by n. The tail estimates are based on the absolute values of the returns in excess of their sample mean; that is the tail index estimates reflect the tail fatness for both the left and right tail simultaneously.
## 2 Results for The Netherlands

This table presents the return and risk statistics based on the Mahieu and van Bussel weighted repeated sales index for Dutch house prices. The CBS Price Index is the general stock market index for the Netherlands. We analyze the monthly log returns on both index values for the period May 1973 though August 1995.

<table>
<thead>
<tr>
<th></th>
<th>Homes</th>
<th>Stock</th>
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<tbody>
<tr>
<td>mean</td>
<td>0.47</td>
<td>0.56</td>
</tr>
<tr>
<td>standard deviation</td>
<td>1.29</td>
<td>4.75</td>
</tr>
</tbody>
</table>

**tail estimates**

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<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>γ</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>s.e. (γ)</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>α</td>
<td>2.82</td>
<td>3.05</td>
</tr>
</tbody>
</table>

The mean returns and the standard deviations on the returns are percentages based on monthly differences between successive log values of the indices. The number of returns used equals 271. The tail estimates are based on the absolute values of the returns in excess of their sample mean; that is the tail index estimates reflect the tail fatness for both the left and right tail simultaneously.